Capacity Utilization, Inflation and Monetary Policy: Marxian models and the New Keynesian Consensus

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February 2005
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Abstract

The paper looks at the adjustment process towards long run equilibrium within a Marxian model provided by Duménil and Lévy (1999), defined in terms of normal rates of capacity utilization. The model is reduced to three essential equations: an IS equation, a Phillips curve equation and a central bank reaction function. It is shown that long run convergence to fully adjusted positions with normal rates of utilization depends on the specific inflation (Phillips curve) equation, and on the central bank setting a zero inflationary target. When these conditions are relaxed, the results are shown to accord more closely with post-Keynesian results. The Marxian model is then contrasted with New Consensus models, the only difference being the latter’s vertical long-run Phillips curve. Such an addition, by contrast, yields fully adjusted positions, and demonstrate a strong isomorphism between these Marxist macro models and the New consensus and New endogenous growth theories – a rather surprising result in the opinion of the authors.

Keywords: monetary policy, central bank, inflation, capacity utilization, New Keynesian, Marxism

JEL classification: E12, E40, E52, E58

* University of Ottawa and University of New South Wales, respectively. We wish to thank Mario Seccareccia and Geoff Harcourt for their helpful comments.
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In an extremely interesting paper, Duménil and Lévy (1999) explore the adjustment mechanism of an economy towards a long-run equilibrium with capacity utilization at normal levels – a fully adjusted position as the Sraffians would call it, or a classical long-term equilibrium as Duménil and Lévy have it. Short-run equilibrium within their model is of the Keynes/Kalecki type, with variability in levels of capacity utilization. One distinctive feature of their model is that it is not the forces of competition which push the economy to a fully adjusted position, but rather aspects of the macro economy coupled with the behaviour of the central bank. In many ways, their analysis of the adjustment process is similar to the analysis of the so-called New Consensus among neoclassical economists (or New Neoclassical Synthesis), which has been defined by a number of New Keynesian economists (such as Romer 2000, Taylor 2000, Walsh 2002, and Woodford 2002), where the return to potential output is achieved through the necessary intervention of the monetary authorities. This paper seeks to look at their underlying framework, comparing it to the New Consensus framework, in order to identify their most essential similarities.

The Duménil and Lévy Model

The important aspect of the model for our purpose, is their specification of the traverse which moves the economy from the Keynesian short-run to the classical long-run equilibrium, defined as a position where “capacity utilization rates are ‘normal’” (Duménil and Lévy 1999, p. 685).

The short run model takes prices as given, and allows capacity utilization to vary. In long run equilibrium, prices are equal to prices of production with uniform rates of profit, and capacity is equal to normal or target rates.
The underlying mechanism driving their traverse is the reaction function of the central bank in the face of inflation. In Duménil and Lévy (1999) the instrument of monetary policy is the central bank’s control over the money supply. However, in section 2.4.3 of their 1999 paper, they make it clear that the analysis of money supply as the appropriate policy instrument is merely a simplification, and that the rate of interest can be readily substituted within their model. Indeed they do this in section 4.4 of their earlier, more extended, draft versions of the published 1999 paper (Duménil and Lévy, 1994, 1997). This is the course we follow, in order to more clearly contrast the model with that of the New Consensus, where it is assumed, as in post-Keynesian models, that monetary control is essentially exercised through discretionary modifications of the interest rate by the monetary authorities. In Duménil and Lévy, monetary policy is adjusted as a result of any actual inflation rate. This, as we shall see, will assure stability of the general price level while “this stability of the general level of prices ensures gravitation of the general level of activity around a normal level” (Duménil and Lévy 1999, p.697).

The reaction function of the central bank is assumed to be:

\[ \Delta r = \beta_1 \pi \]  

Where \( \pi \) is the inflation rate and \( r \) is the rate of interest. All \( \beta \)'s are and will be positive parameters.

In both the 1994 and 1997 drafts, Duménil and Lévy note that they “will not discuss here whether \( r \) should be the nominal or the real rate of interest”. In view of current discussions over monetary policy, it seems clear that the real rate of interest needs to rise whenever price inflation exceeds some target level, so that it is best to interpret \( r \) as a real rate, with \( \beta_1 \) a positive parameter (or else \( r \) is in nominal terms, but \( \beta_1 \) must exceed unity).

As in most macroeconomic models, Duménil and Lévy (equation 16 in both the 1994 and 1997 drafts) assume that investment, or more precisely, the growth rate of capital, is inversely responsive to changes in the rate of interest:

\[ g^i = g_0 - \beta_2 r + \beta_3 u \]  

(2)
where: $g^i$ is the growth rate of capital (the rate of accumulation);
$g_0$ represents the autonomous components of growth;
$u$ is the rate of capacity utilization and
$r$ is the real rate of interest (as long as $g$ is interpreted as the real rate of accumulation).

Equation (2) can be interpreted as a standard IS curve, with an inverse relation between the rate of interest and the level of economic activity $u$, since the equilibrium level of the rate of capacity utilization itself depends positively on the autonomous component of growth and negatively on the rate of interest. The equilibrium rate of capacity utilization can be obtained by confronting the above investment function (equation 2), with the standard classical saving function, which can be written as:

$$g^s = s_c R$$  \hspace{1cm} (3)

where $s_c$ is the propensity to save out of profits and $R$ is the profit rate on capital.

As is well-known the profit rate can be decomposed into three components: $m$ the share of profits (which is a proxy for the strength of the corporate class, through the value taken by the mark-up over wage unit costs), $u$ the rate of capacity utilization, and $v$ the capacity to output ratio, which is assumed to be given by technology. As a result the saving equation can also be rewritten as:

$$g^s = s_c m u / v$$  \hspace{1cm} (4)

This of course implies that the normal rate of profit is $R_n = m u / v$, where $m$ is the assumed exogenous variable that is sorted out by class conflict. The short-run equilibrium rate of capacity utilization, equating equations (2) and (4), is then given by:

$$u = (g_0 - B_2 r) (s_c m / v - B_3)$$  \hspace{1cm} (5)

so that $u_0 = g_0 / (s_c m / v - B_3)$ and $B_4 = B_2 / (s_c m / v - B_3)$ in equation (2), where we have:

$$u = u_0 - B_4 r$$  \hspace{1cm} (6)

So, whenever there is inflation ($\pi > 0$) in the Duménil and Lévy model, real interest rates rise, and hence as a consequence rates of capacity utilization fall. This feedback mechanism will eventually lead to reductions in inflation rates, because Duménil and Lévy (1999) assume that the
inflation rate is a linear function of the discrepancy between the actual rate of capacity utilization, as computed in equation (4), and some normal level of capacity utilization. They have:

$$\pi = \beta (u - u_n)$$  \hspace{1cm} (7)

where: $u$ is still the realised level of capacity utilization and $u_n$ is the optimal level of capacity utilization targeted by enterprises (the normal rate).

Equation (7) represents a standard old-fashioned Phillips curve (it is not a vertical Phillips curve), where there is a straight linear trade-off between the inflation rate and the level of economic activity, without any possibility of shift in the relationship as there are no constant terms. In addition, the equation implies that inflation is nil only when the realized rate of capacity utilization is equal to the normal rate of capacity utilization. The model thus operates in some kind of competitive world, where prices are highly flexible and where they increase whenever demand exceeds normal capacity while they decrease whenever demand falls below normal capacity. This equation, along with equation (1), implies that the rate of interest:

... is constant in a classical long-term equilibrium, since the capacity utilization rate is normal [$u = u_n$] and there is no inflation [$\pi = 0$]........ The coincidence between the absence of inflation and the prevalence of a normal capacity utilization rate is related to the behaviour of enterprises. Because enterprises consider the utilization of productive capacity in the setting of their prices, price stability is associated with a normal capacity utilization rate. .... Within our analysis, prices are a function of disequilibria between supply and demand. (Duménil and Lévy 1999, pp. 698-9)

What happens is that the addition of the central bank reaction function and the inflation mechanism (equations 1 and 7) transforms an otherwise Kaleckian-Keynesian investment function (equation 2) into a classical (Marxist-Sraffian) investment function, where “a deviation of the capacity utilization rate from its normal value would lead to a variation of investment, instead of a constant investment” (Duménil and Lévy 1999, p. 692). Although monetary variables play a role in the determination of the level of economic activity in the short run, according to Duménil and Lévy

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1 This follows from their equation 1, p. 689, and their definition of inflation p. 706.
2 Since we are in a growth model, Duménil and Lévy mean that the rate of accumulation must change.
they have no real effects in the long run. The basic role of monetary variables is to push the economy to its fully-adjusted long run equilibrium, though they play no role in the determination of that equilibrium. In other words, we have the long-term neutrality of money:

Long-term equilibrium can be defined independently of money, but monetary mechanisms are responsible for the convergence of short-term equilibria to long-term equilibrium: they are crucial *vis-à-vis* the stability in dimension of long-term equilibrium. (Duménil and Lévy 1999, p. 710)

The Duménil and Lévy traverse analysis represents an interesting mixture of heterodox and neoclassical theory. As we will see in the next section, the set of equations (1, 6 and 7) is very much reminiscent of the New Consensus model. So is their conclusion that the system will tend to a long-term equilibrium with normal capacity (the equivalent to the NAIRU assumption), with monetary variables having no impact on real variables in the long run.

Another important element of the Duménil and Lévy analysis is their long-term neutrality of money. They argue that, while the monetary system plays a role in pushing the economy to its long-term equilibrium, it does not influence that equilibrium in any way. This has important implications for their underlying story. It is the assumed separation between the forces determining equilibrium and the stability factors pushing the economy to that equilibrium that bring forth their conclusion.

**A modified Keynesian Duménil and Lévy Model**

However, their conclusion needs to be modified if we allow, in the Duménil and Lévy model, the inflation rate target of the central bank to differ from zero, in particular, as is usually the case, to be greater than zero. In this case, the long-term equilibrium will change depending on the inflation target, and therefore on the central bank’s setting of interest rates, so that monetary policy will indirectly influence the long-term equilibrium. This restores the argument of most heterodox

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3 A similar critique of Duménil and Lévy’s approach is provided by Deprez and Dalendina (1994, p. 72).
economists that monetary policy and the monetary system matter even in the long run. The long-term equilibrium position cannot be derived independently of the adjustment path of the economy.

As we will see, the key point of departure with the New Consensus model is the replacement of the latter’s vertical long-run Phillips curve with a more Keynesian Phillips curve, which does allow for long-term trade-offs between inflation and the level of economic activity. Price stability, in their model, is restored due to the reaction function of the central bank. However, their reaction function makes central banks much more rigorous in their anti-inflationary policy than most other commentators would have them. Despite the fact that there is some argument as to the appropriate inflation target (Solow and Taylor 1999), most economists (neoclassical and others) accept that central banks may target a non-zero inflation rate, while Duménil and Lévy impose a target rate of inflation exactly equal to zero. In fact, if a target rate of inflation greater than zero were allowed, then the long-term equilibrium of the economy would be at a level of capacity greater than the normal rate. This follows in a straightforward manner from equation (7), where \( \pi > 0 \Rightarrow u > u_n \).

The heterodox nature of the Duménil and Lévy model, provided some more realistic features are added to it, can be readily seen if we modify equations (1) and (7) by introducing an inflationary target \( \pi^T \) greater than zero in equation (1), writing now:

\[
\Delta r = \beta_3 (\pi - \pi^T) \quad (8)
\]

and by introducing into equation (7) an element of cost inflation \( \pi_c \), based on some institutional or structural features, now rewritten as:

\[
\pi = B_3(u - u_n) + \pi_c \quad (9)
\]

In long run equilibrium, \( \Delta r = 0 \) and \( \pi = \pi^T \). Substituting the value of \( \pi \) taken from equation (9), we obtain the long run value of the actual rate of capacity utilization:

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4 Indeed such a zero-inflation target is tied by Woodford (2002, p. 38) to Knut Wicksell’s proposed rule for setting interest rates.
5 And hence the realized profit rate \( R \) would end up being different from the normal profit rate \( R_n \) in the long run equilibrium.
6 Interestingly, long run capacity utilization lower than normal levels is only possible if a deflationary target is set.
7 As was already suggested in Lavoie (1996, p. 125)
\[ u^* - u_n = (\pi^T - \pi_c)/\beta_5 \]  

(10)

It then becomes obvious that, all else equal, a higher target rate of inflation will be associated with a higher long run rate of capacity utilization and a higher rate of growth. It is equally clear that a classical long run equilibrium, with normal rates of capacity utilization, will only be achieved in the special case where \( \pi^T = \pi_c \), that is when the target rate of inflation set by the central bank will turn out to be equal to cost inflation. In general, this will not necessarily be the case, even if one ventures to suppose that cost inflation ought to be determined in the long run by inflation expectations, which themselves should be anchored by the target inflation rate set by the central bank.

In other words, the reason that the Duménil and Lévy model tends to a specifically classical long term equilibrium is not the underlying nature of the model or the adjustment process. Rather this specific result is achieved as a consequence of the choice of a very peculiar inflation target (\( \pi^T = 0 \)), tied to a very peculiar inflation process (\( \pi_c = 0 \)). This is indeed recognized by Duménil and Lévy (1999, p. 712) themselves when they say that: “Economic policies may direct the system toward targets other than the stability of the general price level …. It would be easy to show in the model that, if such targets are defined, long-term equilibrium will be shifted to another position deviating from the normal utilization of capacity”. In fact, this is true if either condition is modified, as per equations (1B) or (3B). In addition, these modifications mean that monetary policy will not be neutral in the long run. The traverse to the classical long run equilibrium is a very fragile one.

**The “New Consensus” model**

It seems ironic to call this group “new” consensus, as the underlying view of the economy has not changed, in essence since Monetarism Mark 1 associated with Milton Friedman. Like Friedman, New Consensus authors view the long run Phillips curve as being vertical at NAIRU, or at some similar supply-side determined concept, with monetary policy having no impact on real activity in the long run:
There is substantial evidence demonstrating that there is no long-run trade-off between the level of inflation and the level of unused resources in the economy – whether measured by the unemployment rate, the capacity utilization rate, or the deviation of real GDP from potential GDP. Monetary policy is thus neutral in the long run. An increase in money growth will have no long-run impact on the unemployment rate; it will only result in increased inflation. (Taylor 1999 pp. 29-30)

In the short run the inflation rate falls when unemployment is above NAIRU, and increases when unemployment is below it. This is now most often expressed in terms of output gaps – the spread between actual output and potential output – or in terms of capacity utilization. Expressed in terms of capacity utilization, the supply constraint of New Consensus models is given by equation (11):

$$\Delta \pi = \beta_6 (u - u_n)$$

(11)

This is, of course, the vertical long-run Phillips curve, where any deviation of capacity, real GDP or unemployment from their normal levels leads to changes in the inflation rate. If capacity utilization is kept above its normal level, this will quickly lead to accelerating inflation. In other words, unlike equation (7) for Duménil and Lévy, there is no long-term trade-off between inflation and some measure of the output gap.

Like Friedman (and Duménil and Lévy), defenders of the New Consensus view believe that monetary policy can have real effects in the short run as summarized in a conventional IS schedule, so that equation (6) applies as well to the New Consensus model, provided we ignore the additional terms that are included in the more sophisticated versions of the equation (which incorporate expected terms and autonomous shocks, as in Woodford (2002)):

$$u = u_0 - \beta_4 r$$

(6)

Both Friedman and the New Keynesian authors strongly argue that this indicates the need for monetary policy rules. The only difference is that, while for Friedman the rule sets optimal money supply growth, for New Consensus authors “the interest rate rather than the money supply is the key instrument that should be adjusted” (Taylor 1999, p. 47). The proposed rule would have the central bank responding to both price and aggregate demand shocks (or expected such shocks). So,
interest rates should be changed if inflation deviated from its target or if real GDP
deviates from potential GDP. There are many variants of these rules, but the best-known is the so-
called Taylor rule, which, in terms of rates of utilization, is presented as:

\[
i = \pi + \beta_7 (\pi - \pi^T) + \beta_8 (u - u_n) + r_n
\]  

where \(i\) is now a nominal interest rate while \(r_n\) “is the implicit real interest rate in the central
bank’s reaction function” (Taylor 1999, p. 50). We can say that, in Wicksellian terms, it is the
central bank estimate of the ‘natural’ (real) rate of interest.

This can also be rewritten in real terms by taking note that \(i - \pi = r\), so that equation (12A)
becomes equation (12B), which can now more easily be compared to equations (1) and (8):

\[
r - r_n = \beta_7 (\pi - \pi^T) + \beta_8 (u - u_n)
\]  

In the long-term as defined by neoclassical authors, \(\pi = \pi^T\) and \(u = u_n\), so \(r = r_n\). As has been
pointed out by numerous authors, the New Consensus central bank reaction function does not
guarantee that the target inflation rate will ever be achieved however. This is recognized by Taylor
(1999, p. 51) himself, when he says that if the central bank acts on an incorrect estimate of the
natural rate of interest, “then the steady state inflation rate will not equal the target inflation rate”.
The rate of capacity utilization can converge to its normal level without the steady state inflation
rate converging to the target rate set by the central bank. For instance if the implicit real interest rate
estimate is too high, the actual steady state rate of inflation will be too low relative to the target, and
hence the central bank will need to revise downwards its estimate of the ‘natural’ rate of interest.

This problem can be avoided if the central bank reaction function is expressed in difference
terms, as in the modified Duménil and Lévy equation (8) above, or with equation (12C) below.
Indeed, this is how Setterfield (2003, 2004) identifies the typical New Consensus central bank
reaction function:

\[
\Delta r = \beta_7 (\pi - \pi^T) + \beta_8 (u - u_n)
\]  

Setterfield (2003) shows that a model made up of equations (6), (11) and (12C) is always
stable and converges to a normal rate of capacity utilization at the target inflation rate. However, the
second term of the central bank reaction function, given by $\beta_8 (u - u_n)$, plays a crucial role in stability analysis. Without it, the economy would run into a limit cycle, circling the target inflation rate without ever achieving it. What happens is that the second term of equation (12C) provides derivative control, a well-known stabilizing feature since, substituting $(u - u_n)$ by its value in equation (11), we obtain the following reaction function:

$$\Delta r = \beta_7 (\pi - \pi^T) + (\beta_8 / \beta_6)(\Delta \pi) \quad (12D)$$

With equation (12D), the central bank reacts to the level of, and the change in, the inflation rate. In other words, for a given current inflation rate, the central bank would impose a more punitive increase in real interest rates when inflation is quickly rising.

For Duménil and Lévy to be able to recover their conclusion that economists ought to be Keynesian in the short run, but classical in the long run, they need to adopt the three New Consensus equations, namely equations (6) (which they already have), (11) and (12D). With these, monetary policy forces will be such that there is a long-run tendency towards normal rates of capacity utilization, and hence towards fully adjusted positions – the classical long run position – a result which is achieved in addition at the target rate of inflation set by the central bank!

**The similarities with the New Endogenous growth models**

Besides the resemblances with the New consensus on monetary theory, there is also a certain amount of similarity between Duménil and Lévy’s results and those of the so-called New endogenous growth. In the latter, in contrast to the traditional neoclassical Solow growth model, the natural rate of growth is endogenous and achieves higher levels when the economy propensity to save is higher. For instance, in Rebelo (1991), the endogenous growth rate in steady state equals $g^* = sR$, where $s$ is the exogenous average propensity to save while $R$ is the technologically given profit rate on capital (tangible and human).
In Duménil and Lévy (1999, p. 705), the endogenous growth rate in fully adjusted positions, achieved when \( u = u_n \), is given by:

\[
g^* = s_c R_n = s_c m u_n / v
\]

(13)

As in new endogenous growth models, a lower propensity to save \( s_c \) eventually reduces growth rates, and so do higher real wages (or lower normal profit rates \( R_n \)), since they reduce the overall proportion of income that can be saved.

In a short-term equilibrium, a lower saving rate of capitalists has the same effect as a larger real wage, i.e. results in larger capacity utilization rates in the two industries and a larger growth rate, whereas in a long-term equilibrium the profit rate is not affected and a lower saving rate diminishes the growth rate …. (Duménil and Lévy 1999, p. 705).

The interesting feature of the Duménil and Lévy model is that their model, in contrast to the New endogenous canonic growth model (see Dutt 2003), takes the effects of effective demand and class conflict into consideration, as would Keynesian or Kaleckian growth models. However, the inclusion of the specific features of their inflation equation and central bank reaction function produces a traverse towards a long-run classical model, with all the conclusions generally agreed upon by neoclassical authors. For instance, lower propensities to save will eventually generate inflation through standard Keynesian demand-led effects; and hence through the reaction function of the central bank, it will induce the central bank to set higher real interest rates and slower rates of accumulation (as in New endogenous growth models). We thus have a situation where a lower propensity to save leads to higher real interest rates, as in the loanable funds story, and to a slower rates of accumulation, as in New endogenous growth models.

The value taken by the real interest rate in the fully adjusted position can be obtained by substituting \( u \) for its long-run value \( u_n \) in equation (5). We get:

\[
u_n = (g_0 - \beta_2 r) / (s_c m/v - \beta_3)
\]

(14)

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8 This is a standard Marxist result, as can be deduced from Marglin (1984, p. 136). On this, see Lavoie, Rodríguez and Seccareccia (2004).
from which we derive the fully adjusted value of the real rate of interest, or what
Wicksellians would call the natural rate of interest:

\[ r_n = \left( g_0 + \beta_3 u_n - s_c m u_n/v \right)/\beta_2 \]  \hspace{1cm} (14)

It is then obvious that a lower propensity to save \( s_c \) (or a smaller share of profits \( m \)) is
associated with a higher natural rate of interest – the real rate that will be achieved in the long run,
once the fully adjusted position has been reached.

In addition, higher real wages (when assuming no technical progress) may generate
favorable short-run effective demand effects, but in the long run these higher real wages will bring
about a slowdown in accumulation and higher unemployment rates. So once again the iron law of
supply-side accumulation takes over in the long run, as it would in the Rebelo new endogenous
growth model.

**Conclusion**

If we compare the New Consensus model with that of Duménil and Lévy, the similarities are
obvious. Both have the same specification of the IS curve, and identical central bank reaction
functions (save for a target inflation rate of zero). The difference between them hinges mainly on
the specification of the Phillips curve. If a more general specification is included in the Duménil
and Lévy (1999) model, then their conclusion that there is a tendency towards fully adjusted
positions with normal rates of capacity utilization cannot be sustained and long-run
classical/Marxist features cannot be sustained. If we add the standard vertical Phillips curve to the
Duménil and Lévy model, which allow Duménil and Lévy to get in a more general way the results
that they are actually looking for, then we get a perfect New Keynesian model, which includes the
lessons of both the New Consensus on monetary policy and those of New endogenous growth
theory.

We are well aware that the microeconomics of Duménil and Lévy, as they can be found in
detail in Duménil and Lévy (1993), are quite distinct from neoclassical analysis and are quite
appealing, but it remains rather surprising to discover that their macroeconomics appear to be so isomorphic to the most popular models of the New neoclassical synthesis. We have taken the Duménil and Lévy (1999) model as an exemplar, but have little doubt that many other instances could have been found in the literature.

The fact that Marxist and neoclassical models arrive at the same conclusions, of course, does not mean that the former are wrong. But this clearly establishes that if we are on the look for alternative macroeconomic policies (an alternative to received wisdom, as embedded for instance in the Washington consensus), their justification must be found elsewhere than within Marxist macroeconomic theory, which leaves very little scope for change in that regards. By contrast, several post-Keynesian authors deny the validity of the lessons drawn either from the new consensus or from new endogenous growth theory, and thus offer some scope for an alternative. This can be achieved in various ways: by rejecting the vertical long-run Phillips curve, as it has been shown above and by authors such as Setterfield (2004), Hein (2002), Palascio-Vera (forthcoming), or Freedman, Harcourt and Kriesler (2004); by letting the natural rate of growth become endogenous, as in Lavoie (2004); or by letting the normal rate of utilization itself become endogenous, as in Dutt (1997) and Lavoie (1996). Amended models will yield Kaleckian results, with important roles for fiscal and monetary policy in influencing the level of output, capacity utilization and employment.
References


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Appendix

The Duménil and Lévy original model

\[ \Delta r = \beta_1 \pi \]  (1)

\[ u = u_0 - B_4 r \]  (6)

\[ \pi = B_5 (u - u_n) \]  (7)

The Duménil and Lévy revised model

\[ \Delta r = \beta_3 (\pi - \pi^T) \]  (8)

\[ u = u_0 - B_4 r \]  (6)

\[ \pi = B_5 (u - u_n) + \pi_c \]  (9)

The New Consensus model, Mark I

\[ r = r_n + \beta_7 (\pi - \pi^T) + \beta_8 (u - u_n) \]  (12B)

\[ u = u_0 - B_4 r \]  (6)

\[ \Delta \pi = B_6 (u - u_n) \]  (11)

The New Consensus model, Mark II

\[ \Delta r = \beta_7 (\pi - \pi^T) + (\beta_8 / B_6) (\Delta \pi) \]  (12C)

\[ u = u_0 - B_4 r \]  (6)

\[ \Delta \pi = B_6 (u - u_n) \]  (11)

The two-equation differential system then becomes:

\[
\begin{bmatrix}
\Delta r \\
\Delta \pi
\end{bmatrix} =
\begin{bmatrix}
-\beta_8 B_4 & \beta_7 \\
-\beta_6 B_4 & 0
\end{bmatrix}
\begin{bmatrix}
r \\
\pi
\end{bmatrix}
\]

where the trace is \(-\beta_8 B_4 < 0\) while the determinant of the Jacobian is \(+\beta_7 \beta_6 B_4 > 0\), so that the model always converges.